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5.5 Trapezoid Rule

The trapezoid rule is an alternative to using RAMs to approximate area under a curve. We use trapezoids because they can sometimes more accurately approximate the areas under curves than plain old rectangles.

If we have subintervals of equal length, we can use the general Trapezoidal Rule to approximate $\int_a^b f(x)dx$

$$T = \frac{h}{2}\left(y_0 + 2y_1 + 2y_2 + \ldots + 2y_{n-1} + y_n\right)$$

This is equivalent to the average of LRAM and RRAM.

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Ex. 1 Use the Trapezoidal Rule with n=4 to estimate $\int_2^4 2x^2\,dx$

$h = \frac{b-a}{n}$

$h = \frac{4-2}{4}$

$= \frac{1}{2}$

x	2	$\frac{5}{2}$	3	$\frac{7}{2}$	4
y	8	$\frac{25}{2}$	18	$\frac{49}{2}$	32

$T = \frac{\frac{1}{2}}{2}\left(8 + 2\left(\frac{25}{2} + 18 + \frac{49}{2}\right) + 32\right)$

$= \underline{37\frac{1}{2}}$

How does the trapezoidal rule compare with the value of fnint?

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The trapezoidal rule **overestimates** the integral when the integral is concave up and **underestimates** when the graph is concave down.

Ex. 2 An observer measures the outside temperature every hour from noon until midnight, recording temperatures in the following table. Find the average temperature for the 12-hour period.

Time	N	1	2	3	4	5	6	7	8	9	10	11	M
Temp	58	62	64	67	58	76	72	80	59	72	68	56	66

$$h = \frac{b-a}{n} = \frac{12-0}{12} = \underline{\underline{1}}$$

$$T = \int_0^{12} f(x)\,dx = \frac{1}{2}\left[58 + 2(62 + 64 + \ldots + 68 + 56) + 66\right] = 796$$

$$\text{Av temp} = \frac{1}{12} T = \frac{796}{12} \approx \underline{66\tfrac{1}{3} \text{ units}}$$

Sep 21-6:39 PM

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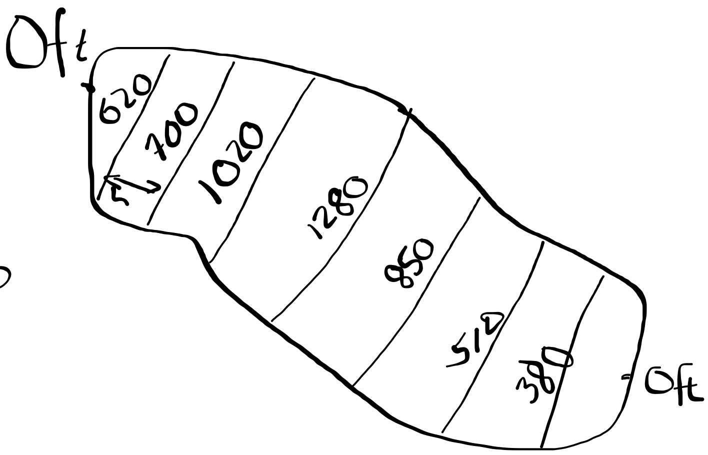
Ex. 3 You are the fish and game keeper for Loch Fyne Estate in Scotland. As such you are responsible for keeping the loch stocked with salmon ready for the fishing season. The average depth of the loch is 60ft. The diagram below shows the distance across the loch measured at 2500-ft intervals.



$h = 2500$

1) Use the Trapezoidal Rule to estimate the volume of the loch.

2) You plan to start the season with two salmon per 100,000 cubic feet. What is the maximum revenue you will generate if the average seasonal catch is 5 salmon per licence and each licence costs £375?

$$A \approx T \quad T = \frac{2500}{2}\left(0 + 2(620 + 700 + \dots + 380) + 0\right)$$
$$\approx 13{,}400{,}000$$

$$Vol = A \cdot depth$$
$$= 13{,}400{,}000 \times 60$$
$$\approx 804{,}000{,}000 \text{ ft}^3$$

16080 fish

$804{,}000{,}000 \div 100000 = 8040$
$\times 2$
$= 16080$

$16080/5 = 3216$

3216 permits $\times 375$

$= £1{,}206{,}000$

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SIMPSON'S RULE

If [a,b] is partitioned into n EVEN subintervals of equal length h=(b-a)/n, we can use the Simpson's Rule to approximate

$$S=\frac{h}{3}(y_0+4y_1+2y_2+4y_3+2y_4+\cdots 2y_{n-2}+4y_{n-1}+y_n)$$

Simpson's is more accurate than the Trapezoidal rule in general because it uses a polynomial approximation to the curve.

Ex. 2 Use Simpson's Rule with n=6 to estimate

$$\int_0^2 \frac{3}{x^3+1}\,dx$$

$h=\frac{2-0}{6}$

$=\underline{1/3}$

x	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2
y	3	$\frac{81}{28}$	$\frac{81}{35}$	$\frac{3}{2}$	$\frac{81}{91}$	$\frac{81}{152}$	$\frac{1}{3}$

$S=\frac{1/3}{3}\left(3+4\left(\frac{81}{28}+\frac{3}{2}+\frac{81}{152}\right)+2\left(\frac{81}{35}+\frac{81}{91}\right)+\frac{1}{3}\right)$

$=\frac{327}{100}$

$=3.27$

Simp $\approx 3.27168134\ldots$

fnint ≈ 3.27

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Error Analysis - Using the approximations for Simpson and the Trapezoidal Rules obviously will incur some form of error.

$$|E_T| \le \frac{b-a}{12} h^2 Mf''$$

$$\frac{(b-a)^3}{12n^2} \quad Mf''$$

$$|E_S| \le \frac{b-a}{180} h^4 Mf^{IV}$$

Mf'' = max value of $|f''|$ on [a,b] – f'' is 2nd deriv
Mf^{IV} = max value of $|f^{IV}|$ on [a,b] - f^{IV} is 4th deriv

h is the biggest contributing factor to these errors. $h < 1$ so $h^4 < h^2$.

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Ex.3

Find the errors using both Trapezoidal and Simpson's Rule for $\int_0^{\pi} \sin x\,dx$ use 8 subintervals.

$f(x) = \sin x$

$f'(x) = \cos x$

$* \; f''(x) = -\sin x$

$f'''(x) = -\cos x$

$* \; f^{iv}(x) = \sin x$

$M_{f''} \to 1$

$$|E_T| \le \frac{\pi - 0}{12}\left(\frac{\pi - 0}{8}\right)^2 \cdot 1$$

$$= \frac{\pi^3}{768} \approx \underline{4.04 \times 10^{-2}}$$

$$|E_S| \le \frac{\pi - 0}{180}\left(\frac{\pi - 0}{8}\right)^4 \cdot 1 \qquad M_{f^{iv}} = 1$$

$$= \frac{\pi^5}{737280} \approx \underline{4.151 \times 10^{-4}}$$

$f_{nint} \approx$

$$|f_{nint} - T| \approx \underline{2.577 \times 10^{-2}}$$

$$|f_{nint} - S| \approx \underline{2.692 \times 10^{-4}}$$